Research on Model Construction and Strategy based on Grey Prediction Model

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Abstract: This paper studies the problem of the best transaction, first carries on the data preprocessing and completes the missing data. Based on the grey prediction model and the improved Markowitz model, the QPDM model is established, and the key index is put forward: based on the established model, RMSE and MAE are introduced as the measurement of the model error. Based on the double error test, we show the test results more intuitively through text descriptions and visual images. The average error rates obtained are all less than 5%, indicating that the model has high accuracy and reflects the superiority of the investment strategy.

1. Introduction

There are two ways to present assets in virtual markets in market transactions, namely gold and bitcoin. Moreover, traders buy and sell these two unstable assets through certain cost assets to maximize their capital and the funds they earn. When making a trade, each trader has a portfolio [C, G, B], which represents the denominations in dollars, troy ounces, and bitcoins in turn. 2021 Contest Dates and Times.

2. Data preprocessing

Gold (Bitcoin) price amount of increase is:

$$f_i = x_i - x_{i-1} \quad i = 2,3,\ldots,n \tag{1}$$

Among them, f_i represents the price increase of gold (bitcoin) on day "i", and represents the price of gold (bitcoin) on day "i".

$$h_i = \frac{x_i - \bar{x}_{15}}{\bar{x}_{15}} \tag{2}$$

Among them, h_i represents gold (bitcoin) BIAS on day "i", x_{15} − represents the average price for the first 15 days.

Bull market evaluation index:

$$\text{Cattle}_i = 0.555 \times \frac{1}{90}\sum_{i=1}^{90} f_i + 0.444 \times \frac{1}{90}\sum_{i=1}^{90} h_i \tag{3}$$

In order to facilitate the calculation later, we normalize the data.

$$g_i = \frac{k_i - k_{\min}}{k_{\max} - k_{\min}} \tag{4}$$

Among them, g_i represents the normalized data which the number of samples is "i", k_i represents the data which the number of samples is "i", $k_{\min}$ represents the maximum value of the sample data, and represents the maximum value of the sample data.

3. The Model of Quantitative Portfolio Decision

(1)Establish a Gray Forecast Model to predict bitcoin and gold's daily price and trend the next day. (2) Using the formula, calculate the composite daily index return from the Predicted daily prices of gold and bitcoin. (3) Deduce the investment value through a multi-objective portfolio decision model based on Transaction Costs and obtain investment value with the help of yield rate of return. According to the predicted results, a reasonable trading strategy is made, whether the trader takes a position, buys, or sells the holdings of bitcoin and gold. (4) Continuously update the investment value in a cycle to obtain the investment value on September 10, 2021.

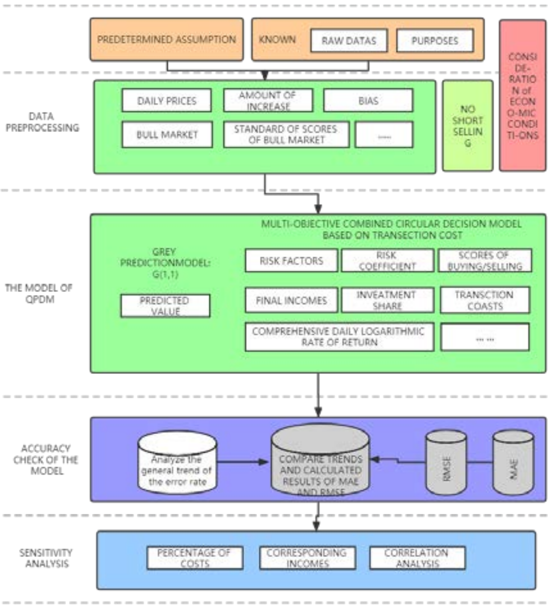

Figure.1 The process of building the model

3.1. GM (1, 1) Model Establishment

Set the raw data sequence:

$$x^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),...,x^{(0)}(m)\right) \tag{5}$$

Get a new sequence (1-IAGO) generated by an accumulation:

$$x^{(1)}=\left(x^{(1)}(1),x^{(1)}(2),...,x^{(1)}(m)\right) \tag{6}$$

And $x^{(1)}(i)=\left\{\sum_{j=1}^{i}x^{(0)}(j)\,|\,i=1,2,......,m\right\}$;

Generating mean sequence:

$$z^{(1)}(i)=\alpha x^{(1)}(i)+(1-\alpha)x^{(1)}(i-1) \qquad k=2,3,...,m \tag{7}$$

In this equation, $0\le\alpha\le1$, this paper takes α =0.5. From this, the grey differential equation is established:

$$x^{(0)}(i)+az^{(1)}(i)=b \qquad i=2,3,...,m \tag{8}$$

The corresponding GM (1,1) whitening differential equation is:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}(t)=b \tag{9}$$

Transform the upwards equation into:

$$-az^{(1)}(i)+b=x^{(0)}(i) \qquad i=2,3,...,m \tag{10}$$

In this equation, a and b are the undetermined model parameters. Equation (10) is expressed in matrix form as:

$$\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ ... & ... \\ -z^{(1)}(m) & 1 \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ ... \\ x^{(0)}(m) \end{pmatrix} \tag{11}$$

It is as same with: $X\beta = Y$

In this equation,

$$X = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ ... & ... \\ -z^{(1)}(m) & 1 \end{bmatrix}, \beta = \begin{pmatrix} a \\ b \end{pmatrix}, Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ ... \\ x^{(0)}(m) \end{pmatrix} \tag{12}$$

After solving equation (11) yields, the least-squares solution is:

$$\overset{\wedge}{\beta}=(a,b)^T=\left(X^TX\right)^{-1}X^TY \tag{13}$$

After solving differential equation (9) yields, the discrete solution of the GM (1,1) model is:

$$\hat{x}(i)=\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-\alpha(i-1)}+\frac{b}{a} \qquad i=2,3,...,m \tag{14}$$

Revert to the original sequence, and get the prediction model:

$$\hat{x}^{(1)}(i)=\hat{x}^{(1)}(i)-\hat{x}^{(1)}(i-1) \quad i=2,3,...,m \tag{15}$$

Taking equation (15) into equation (14), we can finally get:

$$\hat{x}^{(0)}(i)=\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-\alpha(i-1)}(1-e^{\alpha}) \quad i=2,3,...,m \tag{16}$$

According to the meaning, there are two unstable virtual assets in the market of this topic, namely gold and bitcoin. In the grey prediction model, we use $\gamma_1\hat{x}^{(0)}(i)$ and $\gamma_2\hat{x}^{(0)}(i)$ to represents the predicted value of the daily price of gold and bitcoin, respectively.

3.2. Multi-objective Combination Decision-Making Model Based on Transaction Cost

We use the mean to represent the size of the return and the variance, which measures the size of the risk. According to the title, the types of virtual investment products we buy and sell are marked by "n", then n=2. We denote j as the specific type of virtual product. It is stipulated that when "j" is equal to 1, the virtual product traded is gold, while when "j" is equal to 2, the virtual product traded is bitcoin. ""is the yield of "j", x_j is the investment amount of "j", u_j is the upper limit of the investment capital of "j", M_0 is the initial capital of the investor, and the risk coefficient of the return is $\sigma(x)$. As a result, we could know:

$$\sigma(x)=\sqrt{E\left[\sum_{j=1}^{n}R_jx_j-E\left(\sum_{j=1}^{n}R_jx_j\right)\right]^2} \tag{17}$$

Therefore, the portfolio model under the variance risk is:

$$\begin{aligned}&\min\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_jx_j\\&\text{s.t.}\sum_{j=1}^{n}R_jx_j\geq\rho M_0\\&\sum_{j=1}^{n}x_j=M_0\end{aligned} \tag{18}$$

Constraints are "$0\leq x_j\leq$ uj , j=1,2(n).

4. The Improvement of the Model

Set the transaction amount as "x" and the transaction cost function $C(x)$. For example, the literature [4] introduces a typical cost transaction function. According to the actual situation of market transactions, the initial asset amount of market traders is small, and the unit transaction cost is high. Re-buying, the transaction cost will gradually increase, the unit cost will gradually decrease, and the growth trend of the cost function will gradually decrease. Due to the limited set of gold or bitcoin in the market, the virtual capital of the transaction will be reduced when there is a lack of supply. It causes the cost function of a transaction to increase with the transaction's increasing value. We set the transaction cost function as $C(x)$, then

$$C(x)=\left(c_1\left(x_1\right),c_2\left(x_2\right),\ldots,c_n\left(x_n\right)\right) \tag{19}$$

The actual return on the portfolio assets is:

$$y_i=x^{\mathrm{T}}r-\sum_{i=1}^{n}c_i\left(x_i\right)\sim N\left(\mu^{\mathrm{T}}x-\sum_{i=1}^{n}c_i\left(x_i\right);x^{\mathrm{T}}Qx\right) \tag{20}$$

In the state where 2 virtual assets (gold and bitcoin) are available, we obtain the rate of return ""through the above-mentioned gray prediction model. From this, we find that it obeys a normal distribution ". "Among them, ""is the mean vector of returns, and there is an equation ". "The positive definite symmetric matrix ""is the covariance matrix of the random vector ". "

$$x=\left(x_1,x_2,...,x_n\right)^T,x_i\geq 0,i=1,2,...,n,\sum_{i=1}^{n}x_i=1 \tag{21}$$

At the same time, in order to overcome the defect of variance measurement, we introduce the probability of loss to describe the risk ", ""“denotes the profit threshold of market traders, and” $\Pr\left(y\leq\alpha\right)$ ‘denotes the probability of loss of market traders. When the value of ““is as small as possible, the trader’s profit is maximized as much as possible. We assume that the market trader’s expected return is “, “and the objective function ““indicates that the trader’s actual return is greater than his predicted return. To determine whether we are going to buy or sell gold (bitcoin) daily, we introduce the Buying Score formula:

$$p_i=10x_i+5Cattle_i+\frac{1}{\alpha} \tag{22}$$

In order to verify the rationality of the Buying Score, we obtain buying scores of golds and bitcoin to make a visual comparative analysis based on formula (21). According to it, we could obtain Figure 2 and Figure 3.

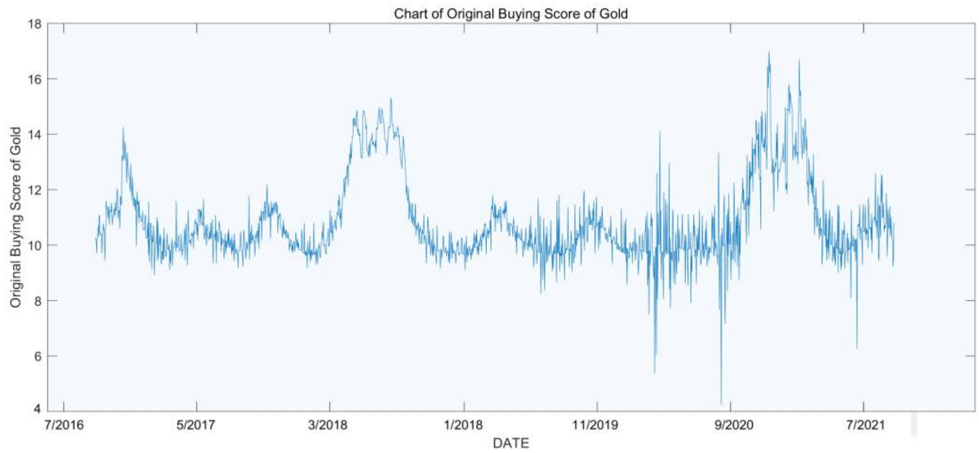

Figure .2 Original buying score of gold

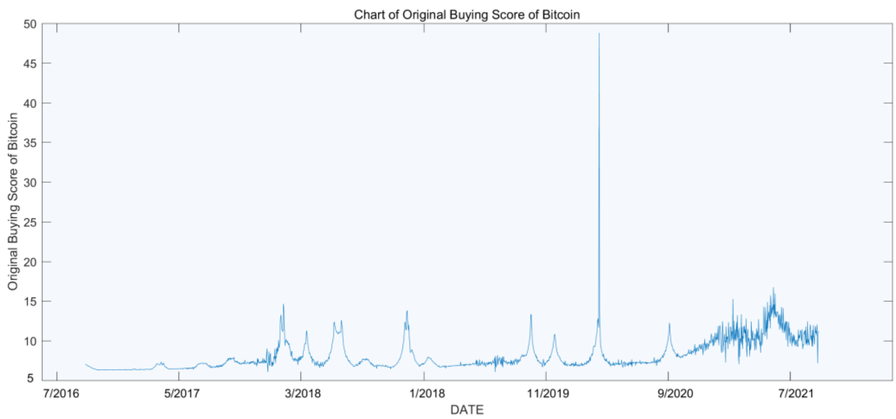

Figure 3. Original buying score of Bitcoin

It is observed that when the buying score of gold increases, the buying score of bitcoin decreases; when the buying score of bitcoin increases, the buying score of gold decreases, so the buy score can be set reasonably.

Normalizing the gold and bitcoin buying scores and visualizing them, we get Figure 4 and Figure 5.

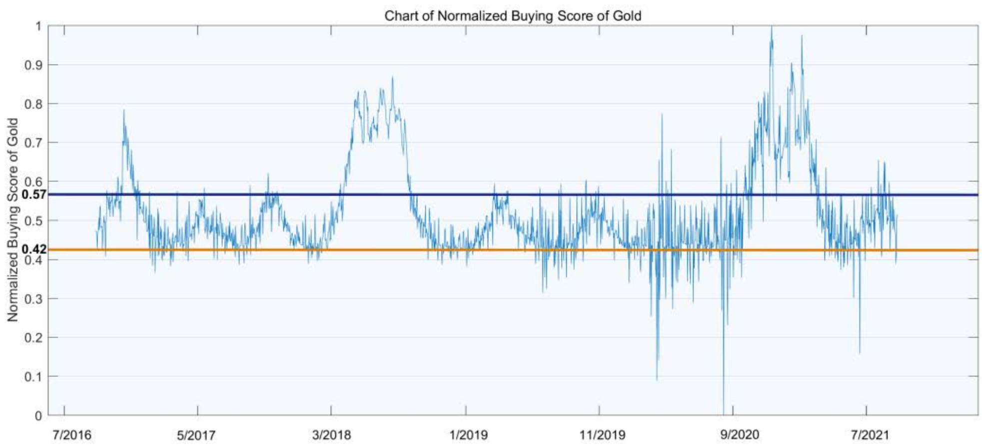

Figure 4. Normalized buying score of gold

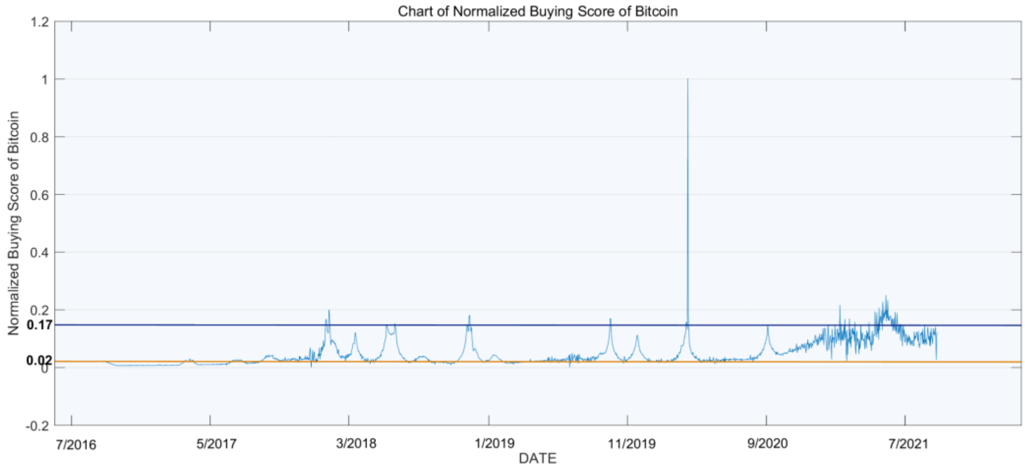

Figure 5. Normalized buying score of Bitcoin

We analyze the visual analysis chart of normalized gold (bitcoin) buying scores and select the densest place of maximum and minimum values to divide the area. The densest place of maximum (minimum) values is the frequent fluctuation area's upper (lower) limit line. You can buy above the upper limit and sell below the lower limit. Meanwhile, you have the choice to hold positions between the upper limit and the lower limit. As shown in Figure 4, gold has an upper limit of 0.57 and a lower limit of 0.42. In Figure 5, bitcoin has an upper limit of 0.17 and a lower limit of 0.02.

5. Conclusions

Gold is a national trading currency, and Bitcoin is a virtual trading product. Traders often get the maximum investment value through the trading of gold and bitcoin. Therefore, this paper first establishes the optimal trading strategy. Based on the grey prediction model and the improved Markowitz model, considering the investment risk and transaction cost, the QPDM model is established, and the key indexes are put forward. The investment strategy is made according to the purchased score and investment shares, and the final investment value of the trader after 5 years is calculated. Based on the double error test, we show the test results more intuitively through text descriptions and visual images. The average error rates obtained are all less than 5%, indicating that the model has high accuracy and reflects the superiority of the investment strategy.

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